

Name: \_\_\_\_\_

Date: \_\_\_\_\_

Period: 6

## Medians and Altitudes of Triangles

on Objective INBAT use and construct points of concurrency of medians and altitudes of  $\Delta$ s.

Median: A median of a triangle is a segment from a vertex to the midpoint of the opposite side

The three medians of the triangle are concurrent. This point of concurrency is called the centroid.

In yesterday's exploration, we saw that the ratio of **larger** : **smaller** segments within the median was 2:1.

Think About It... What will the ratio of the larger segment of the median to the ENTIRE median be? AP:PE

$$AP:AE \rightarrow 2:3$$

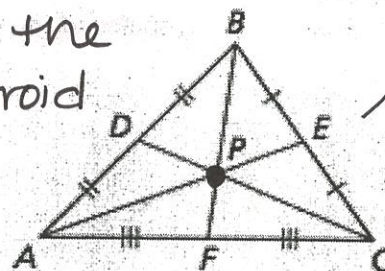
### Centroid Theorem

The centroid of a triangle is  $\frac{2}{3}$  of the distance from each vertex to the midpoint of the opposite side.

The medians of  $\Delta ABC$  meet at point P, and...

$$\frac{AP}{AE} = \frac{2}{3} ; \frac{BP}{BF} = \frac{2}{3} ; \frac{CP}{CD} = \frac{2}{3}$$

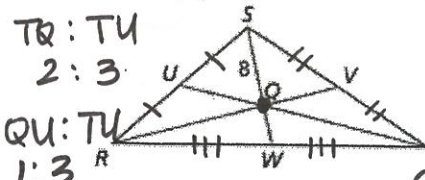
\*P is the centroid



The centroid is also called the "center of gravity" (balance point)

Example:

1. In  $\Delta RST$ ,  $RV = 9$  and  $QU = 2.4$ . Find  $RQ$  and  $TU$ .



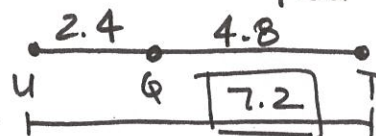
$$\frac{RQ}{RV} = \frac{2}{3}$$

$$9 \left( \frac{RQ}{9} \right) = \left( \frac{2}{3} \right) 9$$

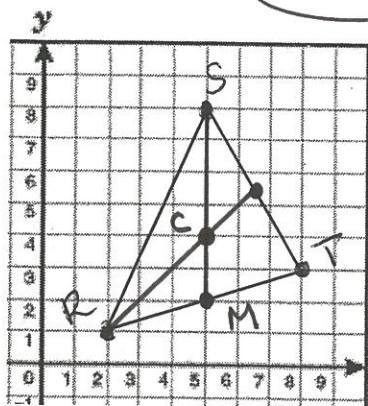
$$RQ = 6$$

$$\frac{QU}{TU} = \frac{1}{3}$$

$$TU = 7.2$$



2. The diagram shows the plan for a triangular piece of a mobile. Where should the sculptor attach the support so that the triangle is balanced?



\*construct median(s)

- ① mdpt of  $\overline{RT}$   
(5, 2)

- ② length of  $\overline{SM} = 6$

- ③  $\frac{2}{3}(6) = 4$ ... count 4 from S

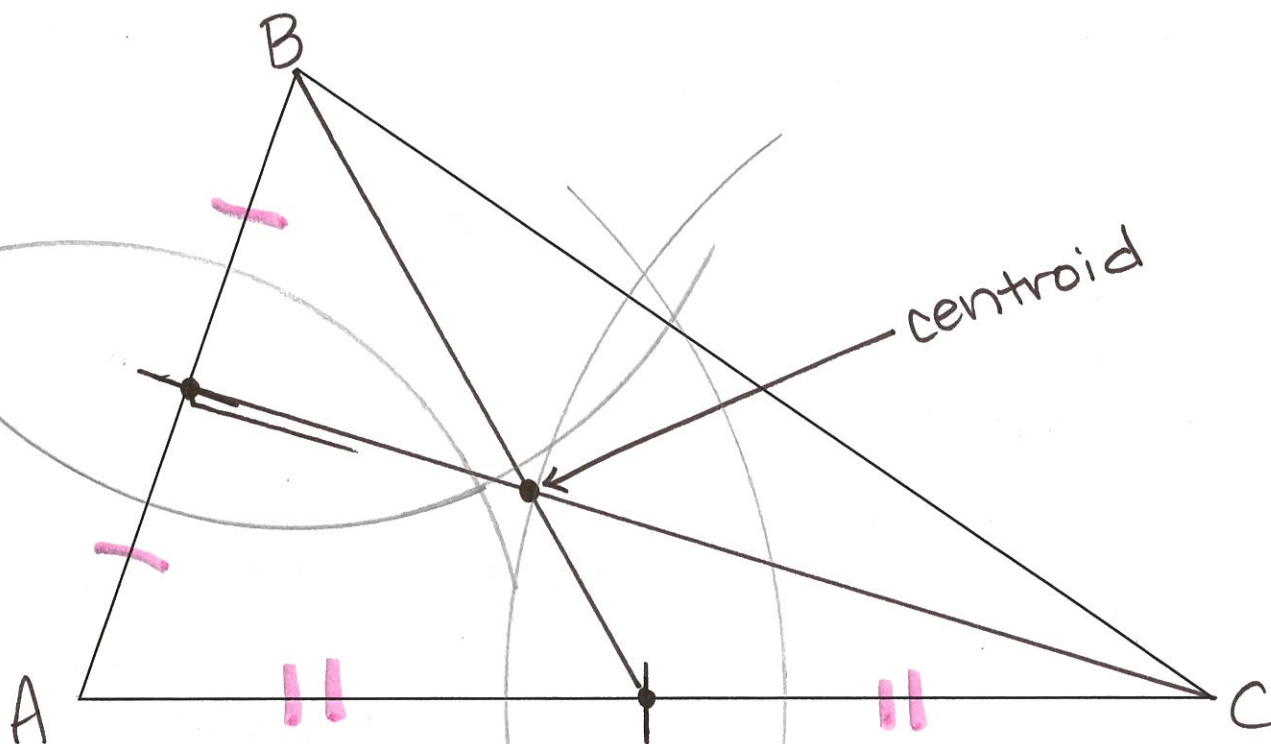
- ② mdpt of  $\overline{ST}$   
(6.5, 5.5)

- ③ where medians intersect

centroid is @ (5, 4)

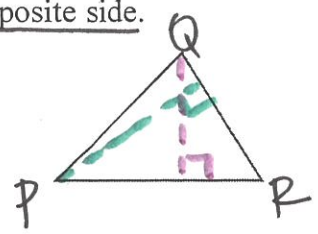
## Construct the Centroid

1. Construct the medians
  - a. Do the construction for the  $\perp$  bisectors
  - b. Use only the midpoint
  - c. Draw the segment from that point to the opposite vertex (remember, the definition for the centroid is the point of concurrency of the medians!)

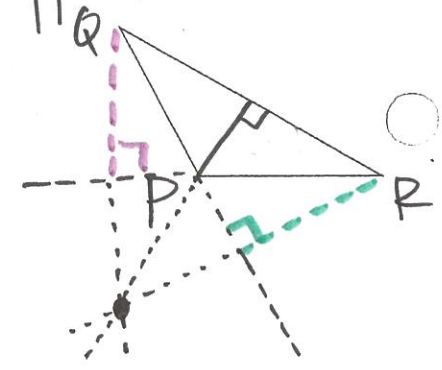


Altitude: An altitude of a triangle is the  $\perp$  segment from a vertex to the opposite side OR the line that contains the opposite side.

obtuse  $\Delta$ s



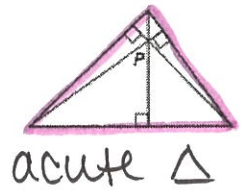
~~Altitude from Q to PR~~



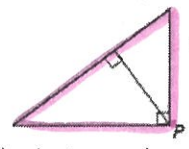
The lines containing the altitudes of a triangle are concurrent.

This point of concurrency is the orthocenter of the triangle.

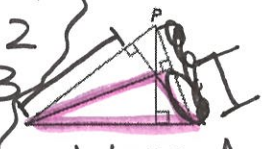
The location of the orthocenter (P, in the diagram below) depends on the type of triangle.



acute  $\Delta$   
(inside  $\Delta$ )



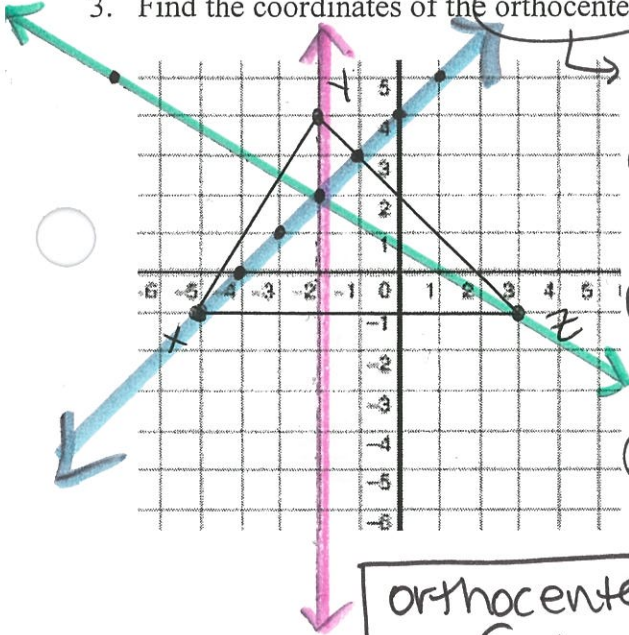
right  $\Delta$   
(on rt.  $\angle$  vertex)



obtuse  $\Delta$   
(outside  $\Delta$ )

Example:

3. Find the coordinates of the orthocenter of  $\Delta XYZ$  with vertices  $X(-5, -1)$ ,  $Y(-2, 4)$  and  $Z(3, -1)$



construct altitudes

altitude from X...

① slope of  $\overline{YZ} = -1$

②  $\perp$  slope to  $\overline{YZ} = 1$

③ count  $\perp$  slope of 1 from X

altitude from Z...

① slope of  $\overline{XY} = \frac{5}{3}$

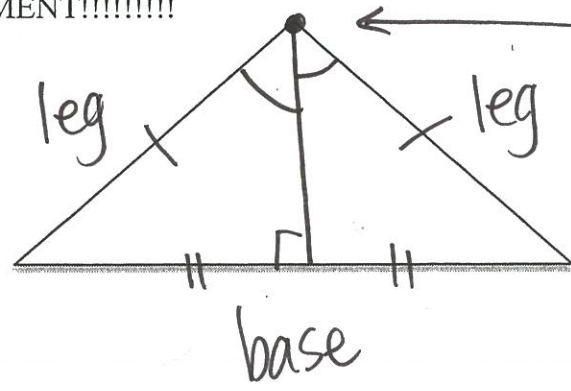
②  $\perp$  slope to  $\overline{XY} = -\frac{3}{5}$

③ count  $\perp$  slope of  $-\frac{3}{5}$  from Z

orthocenter is  
(-2, 2)

In an isosceles triangle, the perpendicular bisector, angle bisector, median, and altitude from the vertex angle to the base are all the SAME SEGMENT!!!!!!!

also for



## Construct the Orthocenter

1. Construct the altitudes
  - a. Do the construction for  $\perp$  line thru point NOT on line
    1. Put anchor on one vertex of the triangle.
    2. Draw an arc that goes through the opposite side TWICE (you may need to extend the opposite side of the triangle to make the arc intersect it two times). Label each point where this arc hits the opposite side.
    3. Construct the  $\perp$  bisector of the segment whose endpoints you just labeled from step #2

